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Exploring basic numerical capacities in children with difficulties in simple arithmetical achievement

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Abstract **Introduction:** Current cognitive theories suggest that mathematical learning disabilities may be caused by a dysfunction in the ability to represent non-symbolic numerosity (non-symbolic skills), by impairments in the ability to associate symbolic numerical representations with the underlying analogic non-numerical magnitude representation (symbolic and numerical mapping skills), or by a combination of both deficits. The aim of this study was to contrast the number sense hypothesis and the access deficit hypothesis, to identify the possible origin of the varying degrees of arithmetical difficulties. **Method:** We compared the performance of children with very low arithmetic achievement (VLA), children with low arithmetical achievement (LA), and typically achieving peers (TA), in non-symbolic, symbolic and numerical mapping tasks. Intellectual capacity and working memory were also evaluated as control variables. The sample comprised 85 Chilean children (3rd to 6th grades) from the Public General Education System. Data were included in several covariance analyses to identify potentially different behavioural profiles between groups. **Results:** The results showed deficits in both non-symbolic numerosity processing and number-magnitude mapping skills in children with VLA, whereas children with LA exhibited deficits in numerical mapping tasks only. **Conclusions:** These findings support the hypothesis of impaired non-symbolic numerical representations as the cognitive foundation of severe arithmetical difficulties. Low arithmetical achievement, in contrast, seems to be better explained by defective numerical mapping skills, which fits the access deficit hypothesis. The results presented here provide new evidence regarding the cognitive mechanisms underlying the different behavioural profiles identified in children with varying degrees of arithmetical difficulties.

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PALABRAS CLAVE:
Dificultades de aprendizaje, sentido numérico, discalculia del desarrollo

Exploración de las capacidades numéricas básicas en niños con dificultades en el rendimiento en aritmética básica

Resumen **Introducción:** Teorías cognitivas actuales sugieren que las dificultades en el aprendizaje de las matemáticas pueden ser causadas por una disfunción en la habilidad de representar las numerosidades no-simbólicas (habilidades no-simbólicas), por dificultades en la habilidad de asociar los números con representaciones analógicas, no-simbólicas, subyacentes a la magnitud (habilidades simbólicas y de mapeo) o por una combinación de ambos déficits. El objetivo de este estudio fue contrastar la hipótesis de un déficit en el sentido del número y la hipótesis del déficit en el acceso, para identificar el posible origen de los diferentes grados de dificultades en aritmética. **Método:** Se comparó el desempeño de niños con muy bajo rendimiento en aritmética (VLA), niños con bajo rendimiento en aritmética (LA) y pares con rendimiento típico (TA), en tareas numéricas no-simbólicas, simbólicas y de mapeo. También se evaluaron la capacidad intelectual y la memoria de trabajo como variables de control. La muestra estuvo conformada por 85 niños chilenos (de 3ero a 6to grado) del Sistema de General de Educación Pública. Los datos fueron incluidos en varios análisis de covarianza para identificar posibles perfiles conductuales diferentes entre grupos. **Resultados:** Los resultados mostraron que los niños con VLA tienen déficits tanto en el procesamiento no-simbólico de la numerosidad como en las habilidades de mapeo entre los símbolos numéricos y la magnitud analógica que estos representan. Los niños con LA solo mostraron déficits en las habilidades de mapeo. **Conclusiones:** Estos hallazgos sustentan la hipótesis de que un daño en las representaciones numéricas no-simbólicas subyace a las dificultades severas en aritmética. Por el contrario, el bajo rendimiento en aritmética parece explicarse por deficientes habilidades de mapeo, lo cual se ajusta mejor a la hipótesis del déficit en el acceso. Los anteriores resultados, ofrecen nuevas evidencias respecto a los mecanismos cognitivos que subyacen a los perfiles conductuales identificados en los niños con diferentes grados de dificultades en aritmética.

During the last decades, multiple studies have been carried out in order to identify the cognitive foundations of mathematical learning disabilities. Some authors have proposed that the origin of the behavioural features exhibited by children with mathematical learning disabilities (including arithmetical difficulties) could be a dysfunction in core numerical non-verbal neurocognitive systems responsible for non-symbolic numerical processing. From this approach, the "number sense" hypothesis states that the origin of mathematical learning disabilities is a deficit in the "approximate number system". According to this hypothesis, difficulties in arithmetical achievement are the consequence of a fundamental difficulty with numerical processing "per se" (Finke et al., 2020; Landerl et al., 2004; Mazzocco et al., 2011; Reigosa-Crespo et al., 2012; Wilson & Dehaene, 2007; Wong et al., 2016, 2017). This model suggests the mental numerical representations of children with arithmetical learning disabilities (ALD) are less accurate compared to that of children with typical mathematical achievement.

On the other hand, the "access deficit" hypothesis (Rousselle & Noël, 2007) proposes that the origin of ALD lies at failure in associating number symbols with their underlying numerical magnitude representations. From this point of view, children with dyscalculia would have intact non-verbal processing systems, but impaired numerical symbols processing due to the ineffective linkage between number symbols and their corresponding analogic numerosity representation. In line with this hypothesis, some studies have shown that children with mathematical learning disabilities differ significantly from controls in symbolic, but not non-symbolic, numerical tasks (Castro & Reigosa, 2011; De Smedt & Gilmore, 2011; Rousselle & Nöel, 2007; Wong & Chan, 2019).

In addition to these domain-specific hypotheses, others have proposed that ALD may be secondary to deficits in domain-general processes, such as executive functions, working memory (WM), and intellectual capacity. Several studies have reported deficits in WM in children with ALD (and also in children with reading disabilities), both when assessing verbal WM with digit span, and when using visuo-spatial span tasks (Aragón et al., 2019; Barnes et al., 2020; Castro et al., 2017; Guzmán et al., 2019; Ibáñez-Azorín et al., 2019).

The current study

Although experimental evidence supports the previously presented hypotheses, the scientific literature also shows inconsistent results, pointing to a research gap regarding the cognitive origin of ALD. This study aimed to compare the performance of children with varying degrees of arithmetical achievement, by contrasting the number sense and the access deficit hypotheses. For this purpose, we evaluated the children in non-symbolic, symbolic and numerical mapping tasks. Few studies analysing the access deficit hypothesis have included paired comparison tasks (in both, symbolic and non-symbolic format). These studies have usually included symbolic comparison or addition tasks, in which the ability to manipulate numerical symbols is properly assessed, but these tasks are not adequate for the assessment of numerical mapping skills. Thus, it is difficult to determine whether the appearance of a "disconnection" between numerical symbols and analogous magnitudes may be caused by a defective interface between the symbolic and analogous magnitude systems or by a deficit in the verbal processing system "per se" (which is implicated in symbolic numerical skills). Hence, it is difficult to determine whether the deficits found in ALD children are due to difficulties in numerical processing, in accessing analogue magnitude through numerical symbols, or if they are associated to deficits in executive functions, intellectual capacity or visuo-spatial abilities. For this reason, intellectual capacity and working memory (verbal and visuo-spatial) capacities were controlled for in all the analyses.

An important source of inconsistency among previous studies regarding the description of ALD pertains to the fact that the authors have used different behavioural effects to describe the typical development of numerical cognition. Some researchers have focused on the numerical ratio, size or distance effects. Different measures and formulas have been used to calculate these effects (using reaction time or accuracy) (Maloney et al., 2010). To avoid the effect of choosing a specific formula to assess numerical mental representations via the ratio, size or distance effects, in this study we used a general efficiency measure (which seizes the relationship between reaction time and accuracy) for assessing general children achievement in numerical tasks.

In this study, we distinguished two groups of children with arithmetical difficulties, depending on their performance in a mental arithmetic task: children with minor arithmetic difficulties (low achievement group: LA); and children with severe arithmetical difficulties (very low achievement group: VLA). Studies that used relatively strict criteria to detect ALD children (e.g., Mazzocco et al., 2011; Wong et al., 2016) have shown that children with severe difficulties in learning arithmetic experience more deep-rooted deficits in their ability to represent and process numerosities (typically named children with developmental dyscalculia). Therefore, the performance of the VLA group in an arithmetic task could be similar to that of children with developmental dyscalculia. Hence, the performance of this group of children could be useful for exploring the cognitive origin of developmental dyscalculia. Note that in this study we have used the term "difficulties" and not "disabilities". The label "disability" connotes a cognitive difference not warranted in studies that do not control for potentially confounding environmental factors (Lewis & Fisher, 2016).

If VLA is a developmental disorder in a core cognitive system underlying the representation and manipulation of numerosities, we hypothesize that children with VLA will show difficulties in all numerical tasks compared to the controls (non-symbolic, symbolic, and numerical mapping skills). In contrast, if VLA is due to a specific problem in accessing the symbolic representation of numbers, they will exhibit difficulties in symbolic and numerical mapping skills, together with impaired analogic magnitude processing, when compared to the controls. Additionally, if VLA emerges as a result of impaired specific-domain (numerical) abilities rather than as a result of domain-general cognitive deficits, the differences in performance in children with VLA compared to the controls should remain, even when controlling for domain-general cognitive processes, such as intellectual capacity and WM capacities.

Method

Participants

The sample comprised 85 Chilean children (50 boys, 3rd to 6th grades) from the Public General Education System; ages ranging between eight years and eight months to thirteen years and eleven months (M = 10.4 years, SD = 1.1 years). Written consent from all parents was obtained, and all participants provided written assent for assessments.

The initial sample selection (103 children) was conducted using the teacher's responses to a questionnaire about risk indicators of difficulties in mathematical achievement. Children without risk indicators for mathematical difficulties were initially included in the typically achieving group. Children with at least one risk indicator for mathematical difficulties were initially identified as children at risk of arithmetical difficulties. Children from either group exhibiting atypical intellectual capacity were excluded from the sample (< 50th percentile on the Raven's Coloured Progressive Matrices Test; Raven et al., 1992). Finally, participants were evaluated using a timed mental arithmetic task (see description below). Similar simple arithmetic tasks for identification of children with ALD have been used in previous studies (Butterworth, 2003; Landerl et al., 2004; Reigosa-Crespo et al., 2012).

Children at risk of arithmetical difficulties were divided into two groups according to their efficiency measure (EM) in mental arithmetic tasks. Thus, the sample was classified in three groups: (1) typical arithmetical achievement (TA or control) group, (2) low arithmetical achievement (LA) group and, (3) very low arithmetical achievement (VLA) group. To distinguish among these groups, we used the Crawford's T-test (Crawford et al., 2010). This T-test was designed for determining neuropsychological deficits by comparing single-case behavioural measures against an appropriate control sample. This method addresses the question of whether individual cases exhibit statistically significant deficits, by treating the control sample statistics as statistics rather than as parameters. For inclusion in the TA group, a *leave-one-out* analysis was conducted to compare each individual mental arithmetic EM to the same school-grade TA group. The individual mental arithmetic EM had to be lower than the mean EM of the corresponding grade group (M) + 1.5 SD. Note, EM is an inverse measure (see the Statistical Analysis section for details); hence, to be classified as TA, individual EM < M + 1.5 SD. To include children at risk of mathematical difficulties in the remaining two groups, individual EMs in the mental arithmetic task were again compared to M of the corresponding TA grade-group's EM. These groups were classified using the following criteria: low achievement group: M + 1.5 SD <= individual EM < M + 3 SD; and, very low achievement group: individual EM > = M + 3 SD. See Table 1 for a detailed sample description.

Materials

Classification task

Timed mental arithmetic task. This task was previously used by Castro et al. (2017). Twenty-eight single-digit additions and 28 single-digit subtractions were presented in two blocks. All items included white Arabic digits (1 to 9) on a black background, presented horizontally in the form "2 + 4". Below each item, two alternative responses, one correct and one incorrect, were simultaneously displayed. Distractors ranged from 1 to 2 units of numerical distance

Table 1 Sample Details

Variables (*SD*)	Groups		
	Typical arithmetical achievement	Low arithmetical achievement	Very low arithmetical achievement
N (boys)	31 (17)	31 (17)	23 (16)
Age	10.3 (*1.1*)	10.4 (*1.2*)	10.5 (*1.0*)
Intellectual capacity	66.0 (*17.9*)	66.3 (*18.1*)	55.4 (*10.5*)*
Verbal WM	4.4 (*1.4*)	4.0 (*1.4*)	3.0 (*1.0*)***
Visuo-spatial WM	38.1 (*7.6*)	26.7 (*10.6*)***	25.7 (*9.4*)***
Efficiency Measure in mental arithmetic task[a]	2311.8 (*797.0*)	5223.7 (*1910.5*) ***	9079.9 (*3328.5*) ***

Note: [a] Efficiency measure is an inverse measure: higher values indicate worse performance. Significant differences compared to TA group: $^{*}p < .05$; $^{***}p < .001$

from the correct answer (resulted from adding or subtracting 1 or 2 to the correct answer). The children had to select the correct answer as quickly as possible, but without making mistakes. Each trial began with the presentation of the stimulus, which remained on screen until the participant offered the answer. Each response was followed by an inter-stimulus interval of 500 ms. Six practice trials were presented before starting the assessment. The reliability coefficient of this task was .87 (subtraction block α = .84; addition block α = .75).

Experimental tasks

Comparison tasks. Two identical blocks of 30 stimuli each (numerosities from 1 to 9) were presented. Comparison pairs varied between two ratios (small number/larger number). Non-symbolic comparison pairs consisted of two sets of dots. To prevent children from relying on perceptual strategies focused on continuous variables, three sets of arrays controlling for density, surface, and area were generated (see a detailed description in Castro et al., 2017). Symbolic comparison pairs consisted of two Arabic digits. The reliability coefficients for these tasks were: symbolic α = .82; non-symbolic α =.90.

Object counting task. This task was used to measure participants' numerical mapping skills since it allows to assess and compare two relatively independent cognitive architectures underlying numerical estimation processes recruited by sets of up to three or four objects (subitizing effect) or larger than 5 objects (counting effect). Two blocks of 30 sets of dots each, were presented (numerosities from 1 to 9, excluding 5). Half of the stimuli corresponded to the subitizing range (numerosities 1 to 4) and the rest to the counting range (numerosities 6 to 9). Children were asked to press the key with the Arabic number corresponding to the number of dots in the array. The reliability coefficient of this task was .82 (subitizing items α = .71, counting items α = .84).

Dot estimation task. This task was used for assessing numerical mapping skills and is similar to the task used by Izard and Dehaene (2007). Children were presented with sets of dots (between 10 and 100 dots) and were instructed to estimate the numerosity. Children were asked to press the keys with the Arabic number corresponding to the approximate number of dots in each array. To prevent children from using perceptual strategies based on continuous variables, three sets of arrays were generated (similar to the three sets of dots for the comparison task). The reliability coefficient of this task was .97.

Number line estimation task. This task is a version of Siegler and Booth's (2004) study and was likewise used for assessing numerical mapping skills. Two identical blocks of 30 stimuli each were presented. Participants were shown a number line with 0 marked on the left end and 100 marked on the right end. Simultaneously, an Arabic numeral was presented above the centre of the number line. Children had to select with the mouse the position in the number line where the Arabic numeral should be located. The reliability coefficient of this task was .87.

For each task, six practice trials were presented before starting the assessment.

Control measures

Nonverbal intellectual capacity. The Raven's Progressive Matrices Test (Raven et al., 1992). A previous study reported a reliability coefficient of .85 for this task (Liporace et al., 2004). The reliability coefficient of this task on this study was .82.

Working memory tasks. Verbal WM was assessed using The Digit Span subtest (backwards) of the Wechsler Intelligence Scale for Children, fourth edition (Taborda et al., 2011). Visuo-spatial WM was assessed using a computerized task. Children were presented with a grid of 20 squares on a white background. Each trial involved presenting a sequence in which grid squares changed colour from white to red, and children had to respond stating which squares changed from white to red, in reverse order to the original sequence. The task consisted of 14 sequences (2, 3, 4, 5, 6, 7 or 8 stimuli, each numerosity was repeated twice). The span score was calculated as the sum of the scores across the 14 trials (see Tillman et al., 2008 for a similar procedure). The reliability coefficients for these tasks were: verbal WM α = .77; visuo-spatial WM α = .75.

Procedure

Children were individually assessed in a quiet room at their school. The experimental tasks were administered in three sessions of 20 - 30 minutes each. In the first session, Raven's Test, WM, and the timed mental arithmetic task were administered. During the second session, comparison tasks and the number line estimation task were administered. Object counting, and dot estimation tasks were administered during the last session.

Statistical Analysis

Achievement in mental arithmetic, comparison and object counting tasks were analysed using efficiency measures (EM) including reaction time (RT) data from correctly answered items. EM were calculated by dividing the median RT of correct responses by the proportion of correct responses. This is an inverse measure (higher efficiency measure represents worse performance) which seizes the relation between RT and accuracy.

Achievement in dot estimation and number line estimation tasks were analysed using the Weber fraction (*w*). We followed the procedure described by Bruandet et al. (2004) to calculate the Weber fraction: *w* = mean of coefficient of variation (CV), where CV = (standard deviation) / (mean of responses for each numerosity).

To test for differences among groups (TA, LA, and VLA) data obtained on numerical tasks (EMs or *w*) were included in different covariance analysis (ANCOVA) with intellectual capacity (IQ) and, verbal and visuo-spatial WM as covariates. Additionally, age was included as a covariate in all these analyses, because the age range of the children in the sample is rather wide (children from 3rd through 6th grade).

Results

Numerical comparison tasks analysis

An ANCOVA was run on the EMs of comparison tasks, with format (non-symbolic, symbolic) as within-subject factor; group (TA, LA and VLA) as between-subjects factor and age, IQ and WM scores as covariates. A statistically significant group effect was found: $F(2, 78) = 13.663$, $p < .001$, $\eta p^2 = .259$, 95% CI TA: [2.941, 3.019]; LA: [3.051, 3.129] and VLA: [3.112, 3.203]. Both, the VLA and LA groups, showed significantly lower performance compared to the TA group ($p < .001$ and $p < .01$, respectively). Additionally, the VLA group performed significantly lower than the LA group ($p < .05$). For the non-symbolic comparison task, planned comparisons showed significant differences between TA and VLA groups ($p < .01$), and a trend towards a significant statistical difference between LA and VLA groups ($p = .06$). No significant differences between the TA and LA groups were found. However, for the symbolic comparison task, planned comparisons showed significant differences between TA and LA ($p < .001$), between TA and VLA ($p < .001$), and between LA and VLA ($p < .001$) groups. No format effect or interaction between format and group were found. See Figure 1.

An additional analysis of comparison of variance for independent samples was performed among groups, by format (symbolic vs. non-symbolic). No significant differences between the groups were found regarding the variability in the non-symbolic comparison task (TA vs. LA: *F*-ratio: 1.982, $p = .07$; TA vs. VLA: *F*-ratio: 1.867, $p = .13$; LA vs. VLA: *F*-ratio: 1.062, $p = .86$). In contrast, a significantly different variability was found in the symbolic comparison task between the TA and LA groups (*F*-ratio: 4.710, $p < .001$) and between the TA and VLA groups (*F*-ratio: 5.086, $p < .001$). LA and VLA groups showed similar variability in the symbolic task (*F*-ratio: 1.080, $p = .83$).

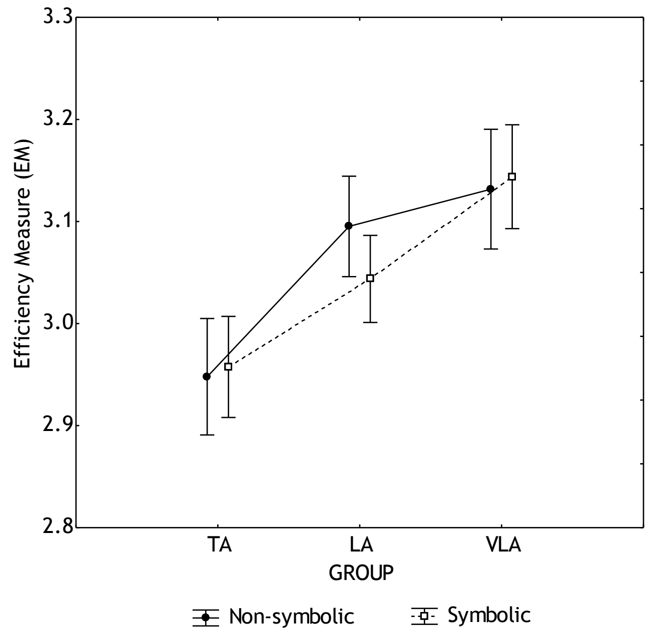

Figure 1. Efficiency measures by groups for numerical comparison tasks: non-symbolic and symbolic. Efficiency measure is an inverse measure: higher values indicate worse performance. TA: Typical arithmetical achievement; LA: Low arithmetical achievement; VLA: Very low arithmetical achievement. The errors bars represent the *SD*.

Object counting task analysis

An ANCOVA was run on counting EMs, with numerical size (small: 1-3 dots and large: 6-8 dots), as within-subject factors; group (TA, LA and VLA) as between-subjects factor and age, IQ and, WM scores as covariates. Sets with 4 dots were excluded from the analysis of the subitizing range because there is controversy concerning individual differences in the subitizing range (up to three or four items). Sets with 9 dots were not included in the analysis to avoid biased responses induced by a ceiling effect.

This analysis showed a statistically significant group effect: $F(2, 78) = 22.803$, $p < .001$, ηp2 = .369, 95% CI TA: [3.275, 3.325]; LA: [3.382, 3.433] and VLA: [3.423, 3.483]. Performance of the VLA group was significantly lower than the TA and LA groups. Also, we found a significant numerical size effect: $F(1, 78) = 14.4746$, $p < .001$, $\eta p^2 = .1567$, 95% CI subitizing: [3.140, 3.179] and counting: [3.597, 3.633]. Finally, an interaction between numerical size and group was found: $F(2, 78) = 10.586$, $p < .001$, $\eta p^2 = .213$, 95% CI [3.084, 3.148] and [3.454, 3.513] for the TA group; [3.126, 3.191] and [3.627, 3.686] for the LA group; [3.166, 3.241] and [3.669, 3.737] for the VLA group (in all cases, the first interval corresponds to the subitizing range and the second interval corresponds to the counting range). Planned comparisons showed no significant differences between TA and LA groups for numerosities in the subitizing range. However, the VLA group's performance was significantly different compared to the TA ($p < .01$) and the LA ($p < .05$) groups. Significant differences between the TA and both arithmetical difficulties' groups were found ($p < .001$ for both) for numerosities in the counting range. A trend towards a significance difference between LA and VLA ($p = .06$) was also found. See Figure 2.

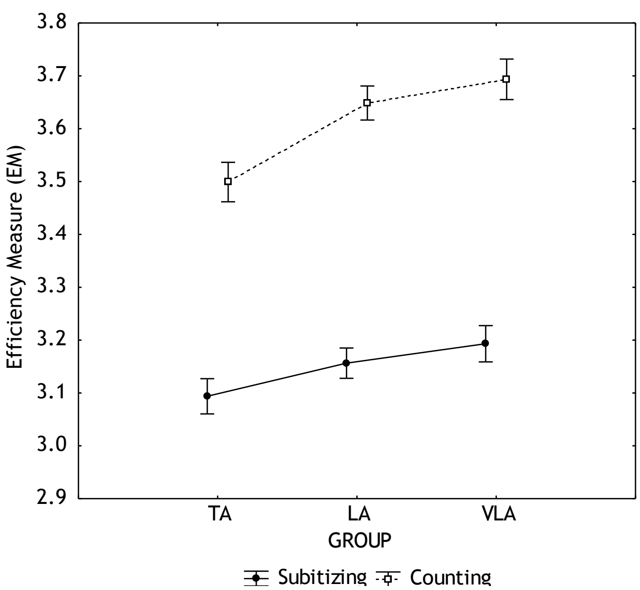

Figure 2. Efficiency measures by groups for object counting task for small (subitizing) and large (counting) numerosities. Efficiency measure is an inverse measure: higher values indicate worse performance.
TA: Typical arithmetical achievement; LA: Low arithmetical achievement; VLA: Very low arithmetical achievement. The errors bars represent the *SD*.

Estimation tasks analysis

An ANCOVA was run on *w* including task as the within-subject factor (dot estimation and number line estimation); group (TA, LA and VLA) as between-subjects factor and; age, IQ and WM scores as covariates. A significant group effect was found: $F(2, 74) = 18.843$, $p < .001$, $\eta p^2 = .337$, 95% CI TA: [.167, .205], LA: [.246, .284] and VLA: [.305, .350]. Planned comparisons showed significant differences between the TA and LA groups for both tasks (dot estimation: $p < .01$; number line estimation: $p < .05$) and, between the TA and VLA groups for both tasks (dot estimation: $p < .001$; number line estimation: $p = .001$). Significant differences ($p < .05$) between the LA and VLA groups were found in the number line estimation task, while achievement in the dot estimation task showed no significant differences between the groups. See Figure 3.

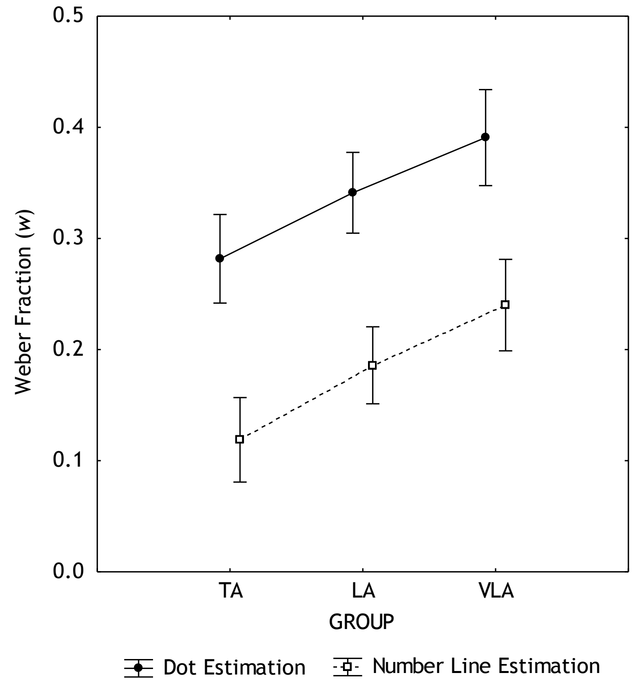

Figure 3. Weber fraction by groups for numerical estimation tasks: dot estimation and number line estimation.
TA: Typical arithmetical achievement; LA: Low arithmetical achievement; VLA: Very low arithmetical achievement. The errors bars represent the *SD*.

Discussion

The aim of this study was to contrast the number sense and the access deficit hypotheses by comparing the performance of children with varying degrees of arithmetical achievement. The LA group showed significant differences compared with TA peers only in the counting and estimation processes, but no differences in non-symbolic comparison nor subitizing were found. These results support the access deficit hypothesis as the cognitive origin of minor difficulties in arithmetical achievement. In contrast, the VLA group showed a significantly lower performance compared to the TA group in all evaluated skills. The VLA group's deficits in number-magnitude mapping might be secondary to the non-symbolic processing deficits. These results support the number sense deficit hypothesis as cognitive origin of severe forms of arithmetical difficulties, which are similar to those showed by children with developmental dyscalculia. These impairments (for both groups) remained when controlling for nonverbal intellectual capacity and verbal and visuo-spatial WM. Therefore, the hypothesis of a dysfunction in domain-general processes as the cognitive foundation of arithmetical learning difficulties was not supported by the present study. Similar differential performance patterns in children with LA and VLA have been previously described by Murphy et al. (2007) and Wong et al. (2017), suggesting a deficit in numerical processing "per se" only in those children exhibiting the most severe difficulties in learning mathematics.

Although, it has been pointed out that basic numerical capacities and its interactions are involved in numerical cognition contributing to mathematical achievement; the different cognitive profiles of children with varying degrees of numerical and arithmetic processing deficits have not been sufficiently characterized, despite its potential relevance to the stimulation of numerical cognition and the design of appropriate intervention strategies. The results presented here provide new evidence supporting the idea that the severity of arithmetic difficulties may also result from different underlying deficits.

Contribution of basic numerical comparison skills (non-symbolic and symbolic) to arithmetical achievement

The significant differences in the non-symbolic comparison task found between the TA and VLA groups support that core numerical cognitive deficits negatively influence arithmetic achievement in VLA children, but are not present in children with low achievement in math (TA and LA groups exhibited similar efficiency levels). In contrast, in the symbolic comparison task, significant differences between TA and the two impaired groups were found; suggesting that numerical symbolic representations and mapping skills involved in associating Arabic symbols to analogue quantity representations, may account for arithmetic deficits both in children with LA and VLA. Previous studies in children with ALD, which used similar tasks, have reported that these children show significantly lower achievement compared to the controls in symbolic tasks, but not in non-symbolic numerical tasks (e.g., De Smedt & Gilmore, 2011; Rousselle & Nöel, 2007). These results are only compatible with the results of our LA group. This inconsistency suggests that the severity of difficulties in arithmetic could be accounted by different underlying difficulties.

Contribution of exact numerosity estimation and numerical mapping skills to arithmetic achievement

It has been suggested that subitizing is a key process for grasping the cardinal meaning of numerals (Hannula et al., 2007; Reigosa-Crespo et al., 2012). On the other hand, object counting is the first systematic procedure that associates number symbols with our underlying analogic/non-symbolic representations of quantity. It allows us to acquire the meaning of these numerals, which is the foundation of more complex mathematical skills (Wong et al., 2017).

Our results show that typical subitizing and counting effects were exhibited by all the groups in the object counting task. However, the VLA group exhibited significant deficits in subitizing compared to the controls and the LA group. Previous studies have shown similar results in children with mathematical learning disabilities (Lafay et al., 2019; Schleifer & Landerl, 2011). Also, we found significant differences between TA and the two groups with arithmetical difficulties in the counting range. VLA (but not LA) children might have to resort to serial counting even for the small numbers' range, resulting in lower efficiency in subitizing. In fact, previous findings showed that dyscalculics count slower than age-matched peers (Landerl et al., 2004) and that slow counters showed worse performance in arithmetic, compared to average and fast counters (Reeve et al., 2012). In line with our VLA group's results, Schleifer and Landerl (2011) found that children with dyscalculia showed steeper slopes in the subitizing range compared to the controls.

The interaction between numerical size (subitizing vs. counting) and group suggests that both, symbolic representations and numerical mapping skills, are key to efficient performance in counting tasks, and that when impaired, they hinder arithmetic achievement. In contrast, exact estimation of numerosities in the subitizing range seems to fundamentally rely on analogic-to-symbolic mapping skills, which, when impaired, are associated with arithmetic dysfluency in children with VLA.

Contribution of approximate numerosity estimation and numerical mapping skills to arithmetical achievement

Regarding approximate estimation, the behavioural signature of this process includes both, a decrease in precision and a linear increase in performance variability with numerosity, following the Weber's Law (Izard & Dehaene, 2007). Usually, the *w* is considered to reflect the resolution of the analogic representations of numerosities, and has been reported to be correlated with mathematical achievement (Mazzocco et al., 2011).

Our results showed that children with VLA exhibited significantly less precise analogic numerical representations compared to controls, but similar *w* compared to the LA group, in the dot estimation task. In the number line task, both groups with arithmetic deficits showed significantly less precise *w* compared to controls. These results show a differential impairment degree, as reflected by the precision of numerical representations corresponding to the different subgroups of children with arithmetic difficulties (LA or VLA). Considering the nature of these estimation tasks (children should translate the non-symbolic quantity to an Arabic number or situate an Arabic number on an analogic line), these results suggest poor mapping skills in both the LA and VLA groups compared to the controls, offering further support to the access deficit hypothesis. Studies using dot estimation and number line tasks, like the ones used in the present study, have reported that children with mathematical disabilities make significantly more errors in their estimations than typically developing peers (Castro & Reigosa, 2011; Geary et al., 2008). Previous studies have systematically shown significant correlations between number estimation tasks and mathematical competence (see the meta-analysis by Schneider et al., 2018).

Conclusions

The results presented here provide new evidence regarding the cognitive mechanisms underlying the different behavioural profiles identified in children with varying degrees of arithmetical difficulties. The study clarifies the relation between non-symbolic and numerical mapping skills in children with low and very low arithmetic achievement. However, considering that sample selection relied on teacher's reports regarding math attainment and a timed mental arithmetic task, future studies should include standardized tests or additional assessments aiming at a more detailed characterization of participants' numerical processing in order to increase the interpretability of these results. Additionally, future studies should describe the developmental trajectories of symbolic processing, including larger sample sizes per grade, and also, children from earlier developmental stages (1st and 2nd grades), when they are starting to master numeric symbols, in order to support the early detection of children at risk of low arithmetical achievement.

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